```
u =
 FullSimplify[{-a (Exp[a x] Sin[a y + d z] + Exp[a z] Cos[a x + d y]) Exp[-d^2 (η / ρ) t],
   -a (Exp[a y] Sin[a z + d x] + Exp[a x] Cos[a y + d z]) Exp[-d^2 (η / ρ) t],
   -a (Exp[a z] Sin[a x + d y] + Exp[a y] Cos[a z + d x]) Exp[-d^2 (η / ρ) t]}];
MatrixForm[u]
```

$$\begin{pmatrix} -a\, e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a z} \cos[a x + d y] + e^{a x} \sin[a y + d z]\right) \\ -a\, e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a x} \cos[a y + d z] + e^{a y} \sin[d x + a z]\right) \\ -a\, e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a y} \cos[d x + a z] + e^{a z} \sin[a x + d y]\right) \end{pmatrix}$$

```
divu = FullSimplify[D[u[[1]], x] + D[u[[2]], y] + D[u[[3]], z]]
```

```
0
```

```
dudt = {D[u[[1]], t], D[u[[2]], t], D[u[[3]], t]};
MatrixForm[dudt]
```

$$\begin{pmatrix} \frac{a d^2 e^{-\frac{d^2 t \eta}{\rho}} \eta \left(e^{a z} \cos[a x+d y]+e^{a x} \sin[a y+d z]\right)}{\rho} \\ \frac{a d^2 e^{-\frac{d^2 t \eta}{\rho}} \eta \left(e^{a x} \cos[a y+d z]+e^{a y} \sin[d x+a z]\right)}{\rho} \\ \frac{a d^2 e^{-\frac{d^2 t \eta}{\rho}} \eta \left(e^{a y} \cos[d x+a z]+e^{a z} \sin[a x+d y]\right)}{\rho} \end{pmatrix}$$

```
gradu = FullSimplify[{{D[u[[1]], x], D[u[[1]], y], D[u[[1]], z]},
    {D[u[[2]], x], D[u[[2]], y], D[u[[2]], z]},
    {D[u[[3]], x], D[u[[3]], y], D[u[[3]], z]}}];
MatrixForm[gradu]
```

$$\begin{pmatrix} a^2 e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a z} \sin[a x + d y] - e^{a x} \sin[a y + d z]\right) & a e^{-\frac{d^2 t \eta}{\rho}} \left(-a e^{a x} \cos[a y + d z] + d e^{a z} \sin[a x + \right. \\ -a e^{-\frac{d^2 t \eta}{\rho}} \left(d e^{a y} \cos[d x + a z] + a e^{a x} \cos[a y + d z]\right) & a^2 e^{-\frac{d^2 t \eta}{\rho}} \left(-e^{a y} \sin[d x + a z] + e^{a x} \sin[a y + d \right. \\ a e^{-\frac{d^2 t \eta}{\rho}} \left(-a e^{a z} \cos[a x + d y] + d e^{a y} \sin[d x + a z]\right) & -a e^{-\frac{d^2 t \eta}{\rho}} \left(d e^{a z} \cos[a x + d y] + a e^{a y} \cos[d x + \right. \end{pmatrix}$$

```
lapu = FullSimplify[{D[u[[1]], x, x] + D[u[[1]], y, y] + D[u[[1]], z, z],
    D[u[[2]], x, x] + D[u[[2]], y, y] + D[u[[2]], z, z],
    D[u[[3]], x, x] + D[u[[3]], y, y] + D[u[[3]], z, z]}];
MatrixForm[lapu]
```

$$\begin{pmatrix} a d^2 e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a z} \cos[a x + d y] + e^{a x} \sin[a y + d z]\right) \\ a d^2 e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a x} \cos[a y + d z] + e^{a y} \sin[d x + a z]\right) \\ a d^2 e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a y} \cos[d x + a z] + e^{a z} \sin[a x + d y]\right) \end{pmatrix}$$

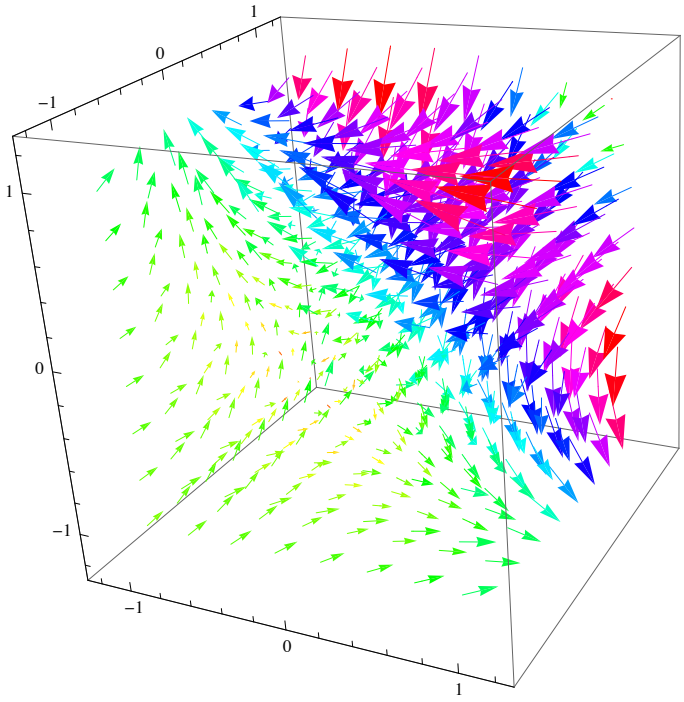
```
g = {0, 0, 0}(*{g1[x, y, t], g2[x, y, t]}*)
```

```
{0, 0, 0}
```

```
MatrixForm[Simplify[u /. {t → 0}]]
```

$$\begin{pmatrix} -a \left(e^{a z} \cos[a x + d y] + e^{a x} \sin[a y + d z]\right) \\ -a \left(e^{a x} \cos[a y + d z] + e^{a y} \sin[d x + a z]\right) \\ -a \left(e^{a y} \cos[d x + a z] + e^{a z} \sin[a x + d y]\right) \end{pmatrix}$$

```
VectorPlot3D[u /. {a → π / 4, d → π / 2, t → 0},
 {x, -1, 1}, {y, -1, 1}, {z, -1, 1}, VectorColorFunction → Hue]
```




```
rhs = Simplify[-ρ (dudt + gradu.u - η / ρ lapu - g)];
MatrixForm[rhs]
```

$$\begin{pmatrix} -a^2\, e^{-\frac{2 d^2 t \eta}{\rho}}\, \rho \left(d\, e^{a (x+y)} \cos[d x + a z] \cos[a y + d z] + a\, e^{2 a x} \cos[a y + d z]^2 + a\, e^{a (x+y)} \cos[a y + d z]\, s \\ -a^2\, e^{-\frac{2 d^2 t \eta}{\rho}}\, \rho \left(a\, e^{2 a y} \cos[d x + a z]^2 + \cos[a x + d y] \left(d\, e^{a (y+z)} \cos[d x + a z] + a\, e^{a (x+z)} \cos[a y + d z]\right)\right. \\ -a^2\, e^{-\frac{2 d^2 t \eta}{\rho}}\, \rho \left(a\, e^{2 a z} \cos[a x + d y]^2 + a\, e^{2 a z} \sin[a x + d y]^2 + a \cos[d x + a z] \left(e^{a (x+y)} \cos[a\right. \end{pmatrix}$$

```
int1 = Simplify[Integrate[rhs[[1]] , x]]
```

$$-\frac{1}{2} a^2\, e^{-\frac{2 d^2 t \eta}{\rho}}\, \rho \left(e^{2 a x} + e^{a (y+z)} \sin[d (-x + y) + a (x - z)] - e^{a (x+z)} \sin[a x - a y + d y - d z] + e^{a (x+y)} \sin[d x - a y + a z - d z] + e^{a (y+z)} \sin[d (x + y) + a (x + z)] + e^{a (x+y)} \sin[d (x + z) + a (y + z)] + e^{a (x+z)} \sin[a (x + y) + d (y + z)]\right)$$

```
int2 = Simplify[Integrate[rhs[[2]] , y]]
```

$$-\frac{1}{2} a^2\, e^{-\frac{2 d^2 t \eta}{\rho}}\, \rho \left(e^{2 a y} + e^{a (y+z)} \sin[d (-x + y) + a (x - z)] - e^{a (x+z)} \sin[a x - a y + d y - d z] + e^{a (x+y)} \sin[d x - a y + a z - d z] + e^{a (y+z)} \sin[d (x + y) + a (x + z)] + e^{a (x+y)} \sin[d (x + z) + a (y + z)] + e^{a (x+z)} \sin[a (x + y) + d (y + z)]\right)$$

```
int3 = FullSimplify[Integrate[rhs[[3]] , z]]
```

$$-\frac{1}{2} a^2\, e^{-\frac{2 d^2 t \eta}{\rho}}\, \rho \left(e^{2 a z} + e^{a (y+z)} \sin[d (-x + y) + a (x - z)] + 2\, e^{a (x+y)} \cos[a y + d z] \sin[d x + a z] + 2\, e^{a (x+z)} \cos[a x + d y] \sin[a y + d z] + e^{a (y+z)} \sin[d (x + y) + a (x + z)]\right)$$

```
p = FullSimplify[-1/2 a^2 E^(-(2 d^2 t η)/ρ) ρ
    (E^(2 a x) + E^(2 a y) + E^(2 a z) + E^(a (y+z)) Sin[d (-x + y) + a (x - z)] - E^(a (x+z)) Sin[a x - a y + d y - d z] +
      E^(a (x+y)) Sin[d x - a y + a z - d z] + E^(a (y+z)) Sin[d (x + y) + a (x + z)] +
      E^(a (x+y)) Sin[d (x + z) + a (y + z)] + E^(a (x+z)) Sin[a (x + y) + d (y + z)])]
```

$$-\frac{1}{2} a^2 e^{-\frac{2 d^2 t \eta}{\rho}} \rho \left(e^{2 a x} + e^{2 a y} + e^{2 a z} + 2 e^{a (y+z)} \text{Cos}[d x + a z] \text{Sin}[a x + d y] + 2 e^{a (x+y)} \text{Cos}[a y + d z] \text{Sin}[d x + a z] + 2 e^{a (x+z)} \text{Cos}[a x + d y] \text{Sin}[a y + d z]\right)$$

```
gradp = {D[p, x], D[p, y], D[p, z]};

Simplify[gradp[[1]] - rhs[[1]]]
```

0

```
Simplify[gradp[[2]] - rhs[[2]]]
```

0

```
Simplify[gradp[[3]] - rhs[[3]]]
```

0

```
MatrixForm[Simplify[dudt + gradu.u + 1 / ρ gradp - η / ρ lapu - g]]
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
u0 = u /. {t → 0, x → x[0], y → x[1], z → x[2]}
```

$$\left\{-a \left(e^{a x[2]} \text{Cos}[a x[0] + d x[1]] + e^{a x[0]} \text{Sin}[a x[1] + d x[2]]\right), -a \left(e^{a x[0]} \text{Cos}[a x[1] + d x[2]] + e^{a x[1]} \text{Sin}[d x[0] + a x[2]]\right), -a \left(e^{a x[1]} \text{Cos}[d x[0] + a x[2]] + e^{a x[2]} \text{Sin}[a x[0] + d x[1]]\right)\right\}$$

```
CForm[u0[[3]]]
```

```
-(a*(Power(E,a*x(1))*Cos(d*x(0) + a*x(2)) + Power(E,a*x(2))*Sin(a*x(0) + d*x(1))))
```

```
p0 = p /. {t → 0, x → x[0], y → x[1], z → x[2]}
```

$$-\frac{1}{2} a^2 \rho \left(e^{2 a x[0]} + e^{2 a x[1]} + e^{2 a x[2]} + 2 e^{a (x[1]+x[2])} \text{Cos}[d x[0] + a x[2]] \text{Sin}[a x[0] + d x[1]] + 2 e^{a (x[0]+x[1])} \text{Cos}[a x[1] + d x[2]] \text{Sin}[d x[0] + a x[2]] + 2 e^{a (x[0]+x[2])} \text{Cos}[a x[0] + d x[1]] \text{Sin}[a x[1] + d x[2]]\right)$$

```
ut = u /. {x → x[0], y → x[1], z → x[2]}
```

$$\left\{-a e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a x[2]} \text{Cos}[a x[0] + d x[1]] + e^{a x[0]} \text{Sin}[a x[1] + d x[2]]\right), -a e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a x[0]} \text{Cos}[a x[1] + d x[2]] + e^{a x[1]} \text{Sin}[d x[0] + a x[2]]\right), -a e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a x[1]} \text{Cos}[d x[0] + a x[2]] + e^{a x[2]} \text{Sin}[a x[0] + d x[1]]\right)\right\}$$

```
ut[[3]]
```

$$-a e^{-\frac{d^2 t \eta}{\rho}} \left(e^{a x[1]} \text{Cos}[d x[0] + a x[2]] + e^{a x[2]} \text{Sin}[a x[0] + d x[1]]\right)$$